

Interaction and Evolution of Two-Colour Nematicons in the Local Response Regime

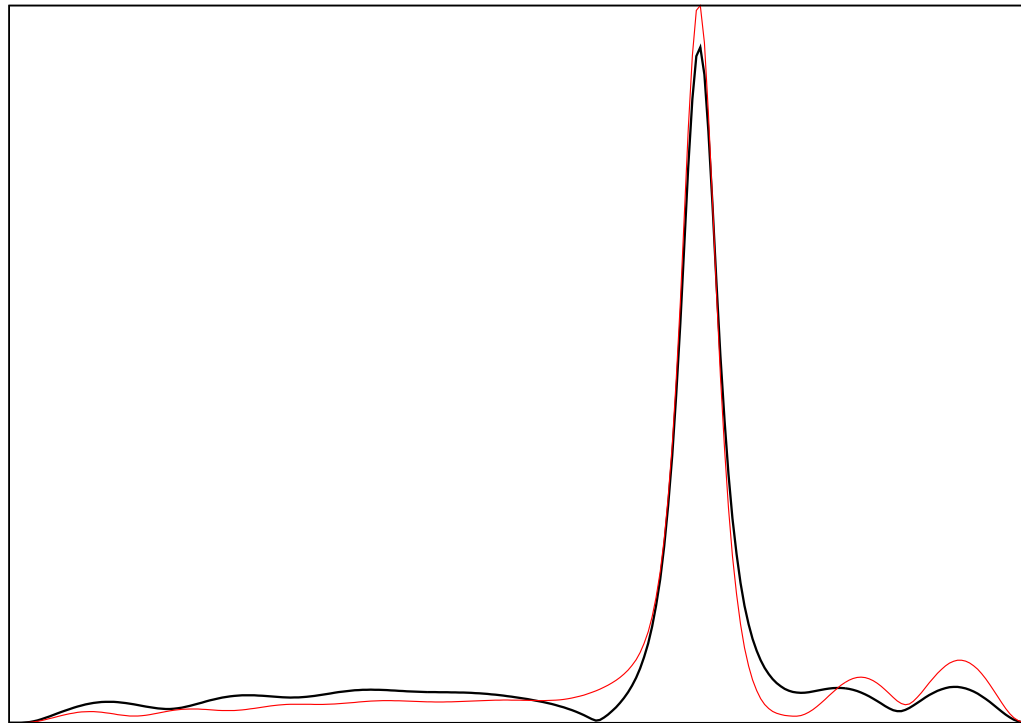
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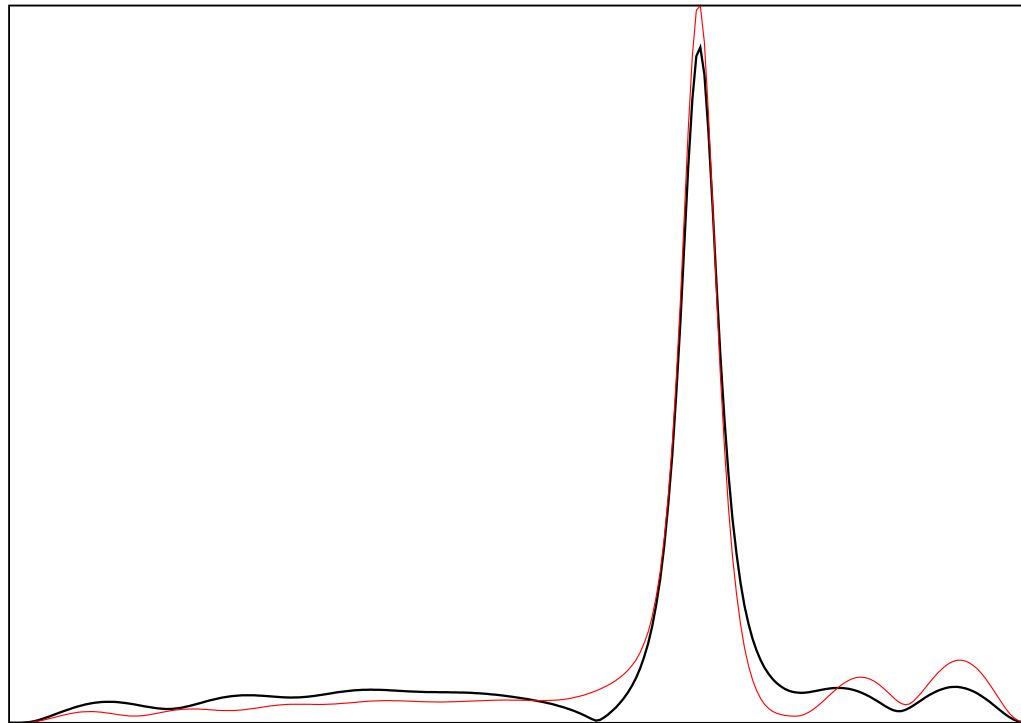
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Two-Colour Nematicons



■ Nematic Liquid Crystals

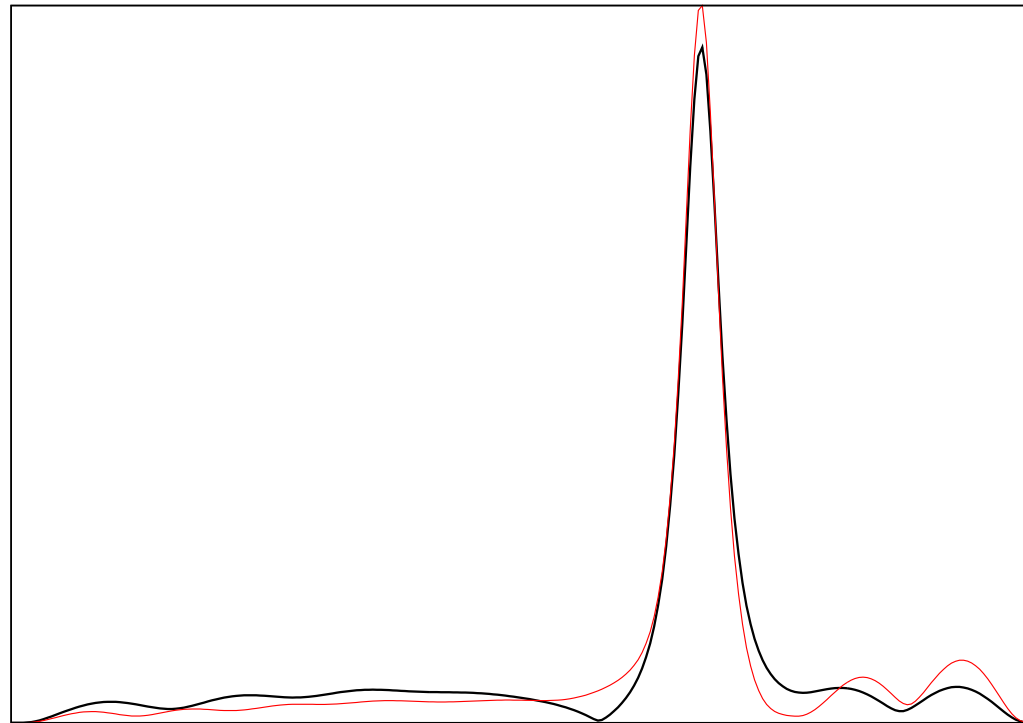
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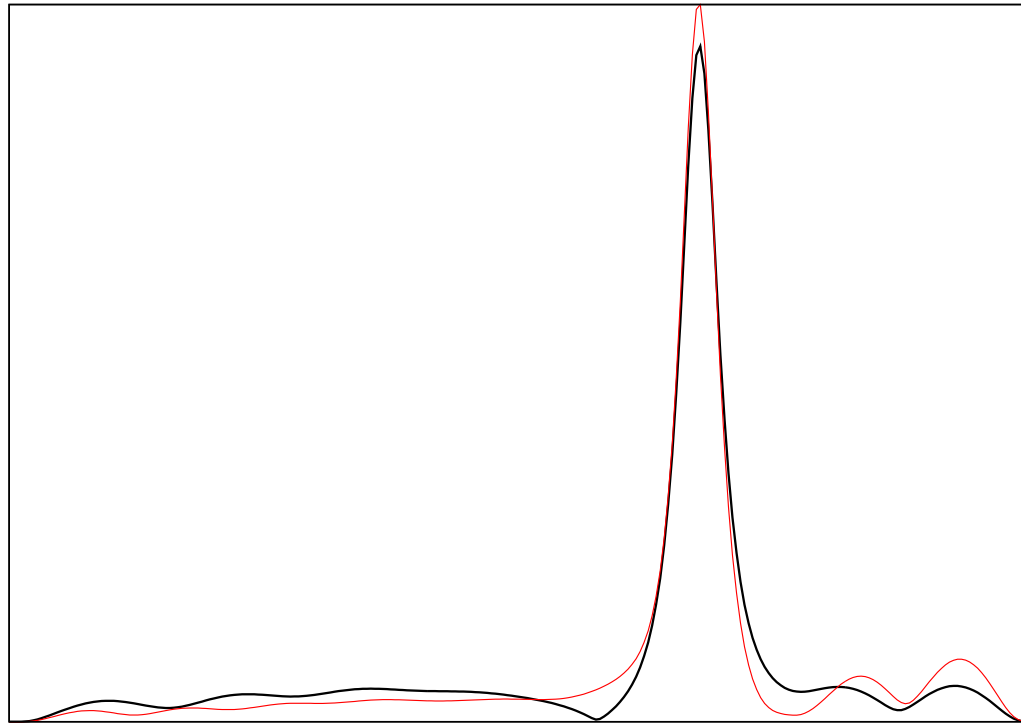
■ Experimental Results

Two-Colour Nematicons



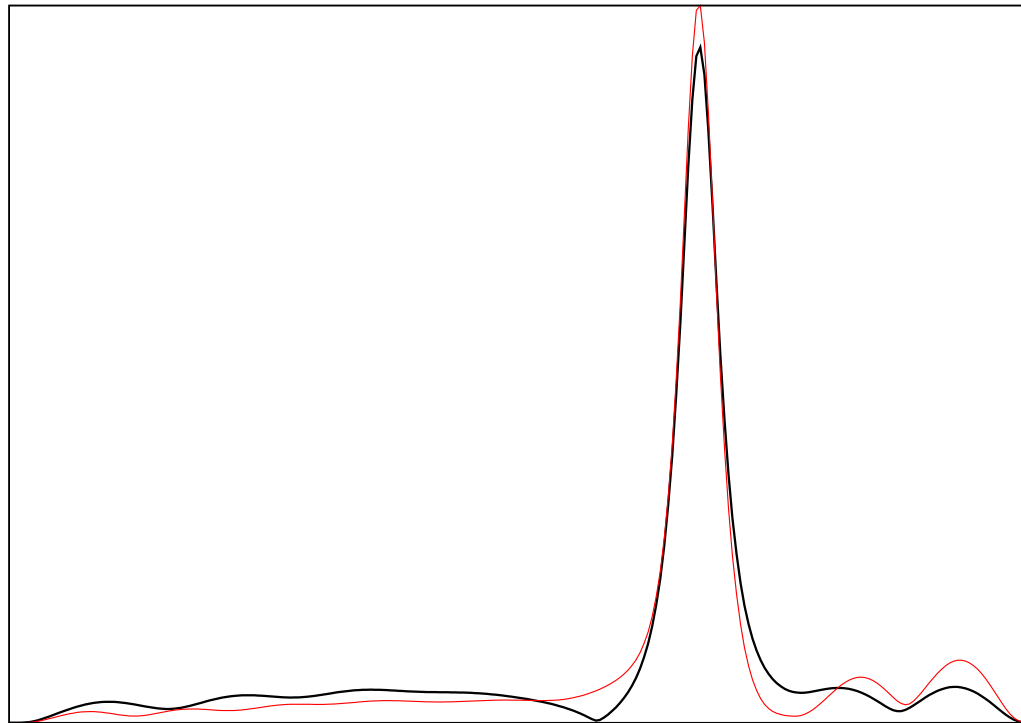
- Nematic Liquid Crystals
- Experimental Results
- Governing Equations

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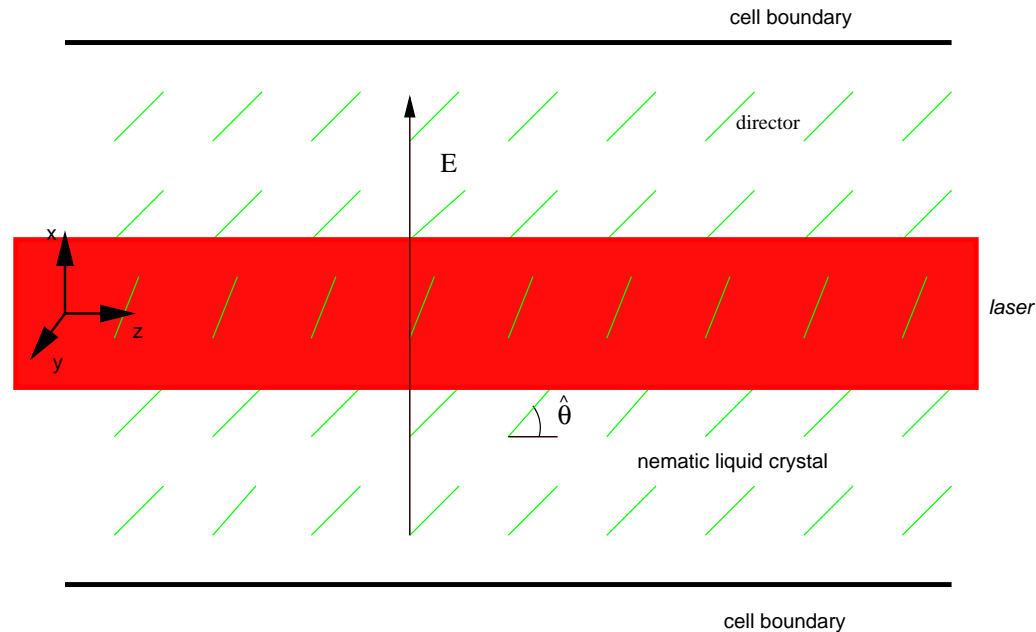
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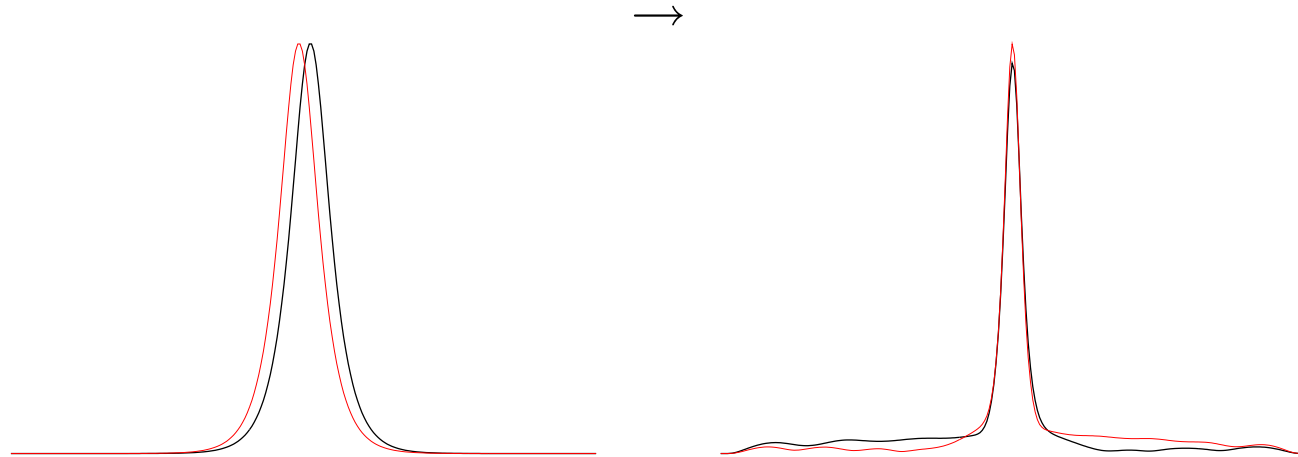
NLCs and Nematicons



- Nematic liquid crystal: rod-like molecules.
- Molecules tend to re-orientate in direction of electric field.
- Coherent, polarised optical beam.
- Beam intensity reduces angle of nematic molecules w.r.t. electric field.
- Beam self focuses.

Two-Colour Nematicons

- Different wavelengths \Rightarrow different degrees of birefringence and dispersion.
- Cross-phase modulation (CPM) negates this effect.
- Self-localised VS forms.



Governing Equations



■ Single Nematicon Governing Equations

- NLS-like equation for electric field of the light.
- Forced heat equation for the director.

■ Two-Colour Nematicon Governing Equations

- CNLS-like equation for each colour

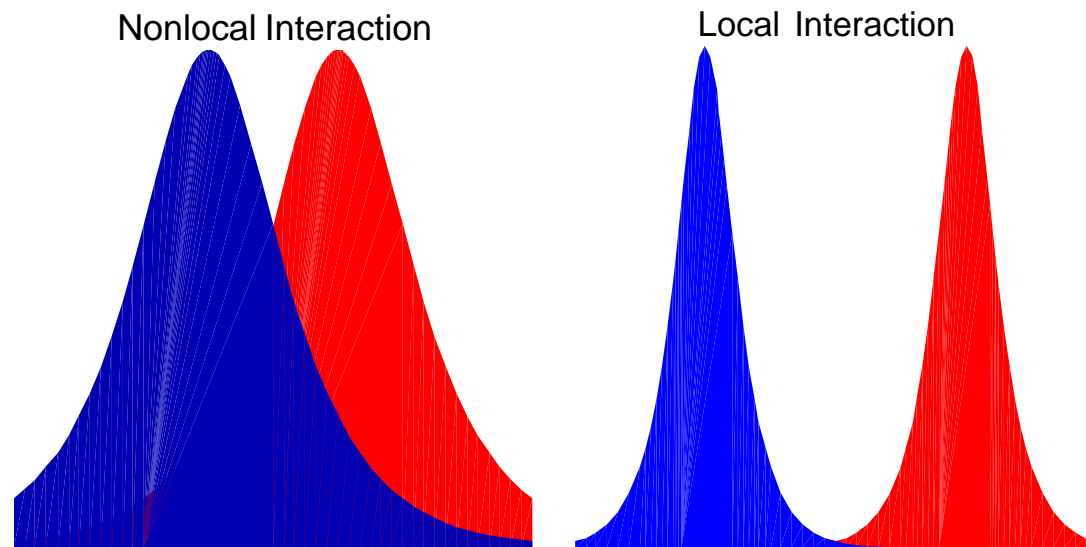
$$iu_t + \frac{D_1}{2} \nabla^2 u + Au \sin 2\theta = 0$$
$$iv_t + \frac{D_2}{2} \nabla^2 v + Bv \sin 2\theta = 0.$$

- Director equation

$$q \sin 2\theta - 2A|u|^2 \cos 2\theta - 2B|v|^2 \cos 2\theta = \nu \nabla^2 \theta$$

ν large and ν small

- ν : ratio of elastic energy of nematicon to energy of electric field. Nonlocality parameter.



ν large and ν small

- ν : ratio of elastic energy of nematicon to energy of electric field. Nonlocality parameter.
- $\nu \rightarrow \infty$ for nonlocal crystals.
 - Low power input beam. Studied experimentally extensively.
 - Nonlocality stops collapse.
- $\nu \rightarrow 0$ for local crystals.
 - Strongly illuminated or heated NLCs.
 - Stability caused by saturation of NLC director motion.

ν small

Consider director equation for ν small <-

Rearrange

$$\tan 2\theta = \frac{2(A|u|^2 + B|v|^2)}{q}$$

Assuming radial symmetry, the governing equations are

$$iu_t + \frac{D_1}{2}u_{rr} + \frac{D_1}{2r}u_r + \boxed{Au \left(\frac{2(A|u|^2 + B|v|^2)}{\sqrt{q^2 + 4(A|u|^2 + B|v|^2)}} \right)} = 0$$

$$iv_t + \frac{D_2}{2}v_{rr} + \frac{D_2}{2r}v_r + \boxed{Bv \left(\frac{2(A|u|^2 + B|v|^2)}{\sqrt{q^2 + 4(A|u|^2 + B|v|^2)}} \right)} = 0.$$

Saturable nonlinearity

Modulation Equations

The Lagrangian for these equations is

$$L = ir (\bar{u}u_t - u\bar{u}_t) + ir (\bar{v}v_t - v\bar{v}_t) - D_1 r |u_r|^2 - D_2 r |v_r|^2 + r \sqrt{q^2 + 4(A|u|^2 + B|v|^2)} - rq.$$

Next steps:

- Insert trial functions into Lagrangian.

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- Plus various conservation equations.

Nematicon Equations

Trial Functions

$$u = a_1 \operatorname{sech} \left(\frac{\sqrt{(x - \xi_1)^2 + y^2}}{w_1} \right) e^{i\sigma_1 + iV_1(x - \xi_1)} + ig_1 e^{i\sigma_1 + iV_1(x - \xi_1)}$$

$$v = a_2 \operatorname{sech} \left(\frac{\sqrt{(x - \xi_2)^2 + y^2}}{w_2} \right) e^{i\sigma_2 + iV_2(x - \xi_2)} + ig_2 e^{i\sigma_2 + iV_2(x - \xi_2)}.$$

Alternative trial functions

$$u = a_1 e^{-\frac{(x - \xi_1)^2 + y^2}{w_1^2}} e^{i\sigma_1 + iV_1(x - \xi_1)} + ig_1 e^{i\sigma_1 + iV_1(x - \xi_1)}$$

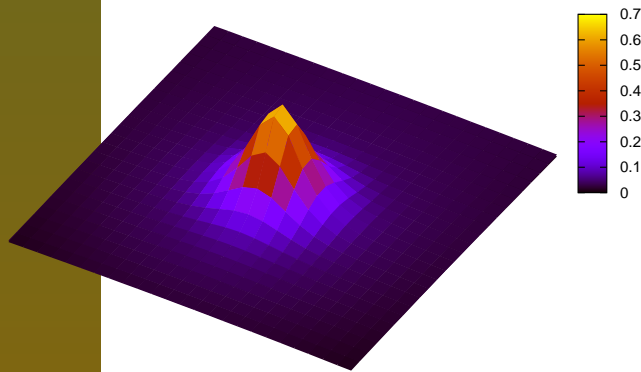
$$v = a_2 e^{-\frac{(x - \xi_2)^2 + y^2}{w_2^2}} e^{i\sigma_2 + iV_2(x - \xi_2)} + ig_2 e^{i\sigma_2 + iV_2(x - \xi_2)}.$$

Nematicon Equations

- The first terms represent varying soliton-like pulses.
- Second terms represent low wavenumber, low phase-speed radiation shed by the pulses. Should form circular shelves under the pulses.

where

- a_k : amplitude
- w_k : width
- ξ_k : displacement from x
- σ_k : phase
- V_k :
- g_k : shelf amplitude



Equivalent Gaussians

- CPM integrals for the averaged Lagrangian cannot be evaluated as they are nonlinear terms involving sech. eg.

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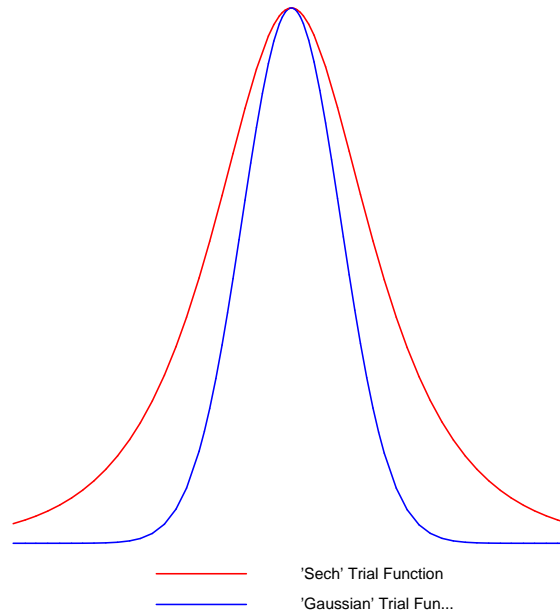
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Radiation Loss

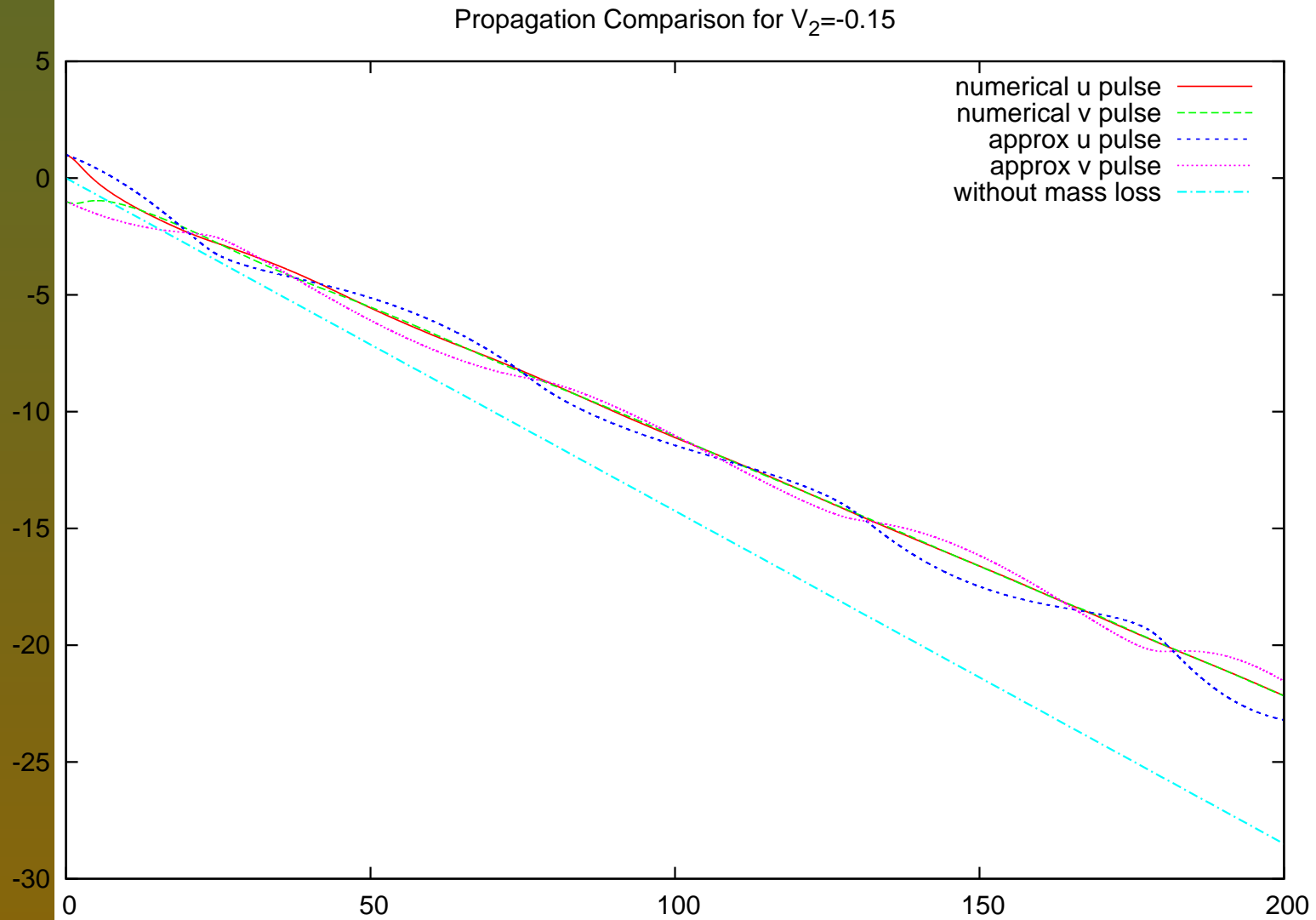
- Shed radiation has small amplitude relative to the nematicons.
- Governed by **linearised** NLS equations

$$iu_z + \frac{D_1}{2r} \frac{\partial}{\partial r} (ru_r) = 0$$

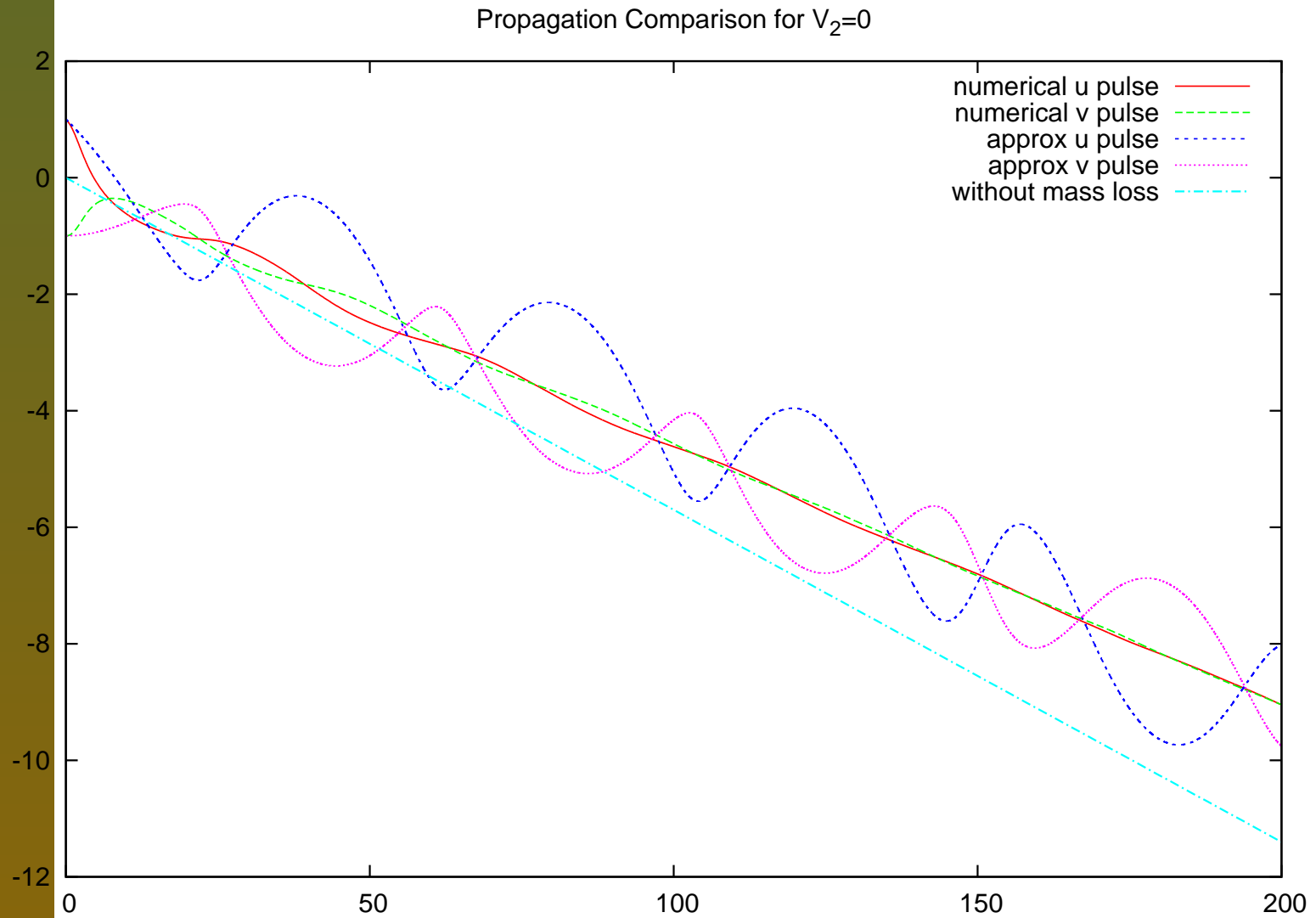
$$iv_z + \frac{D_2}{2r} \frac{\partial}{\partial r} (rv_r) = 0$$

- Match shed radiation to existing radiation value at edge of shelf.
- Mass lost to the shed radiation important.
- Mass loss and radiation shelf are both initially 0.
- Add mass loss term to equation for g_i .

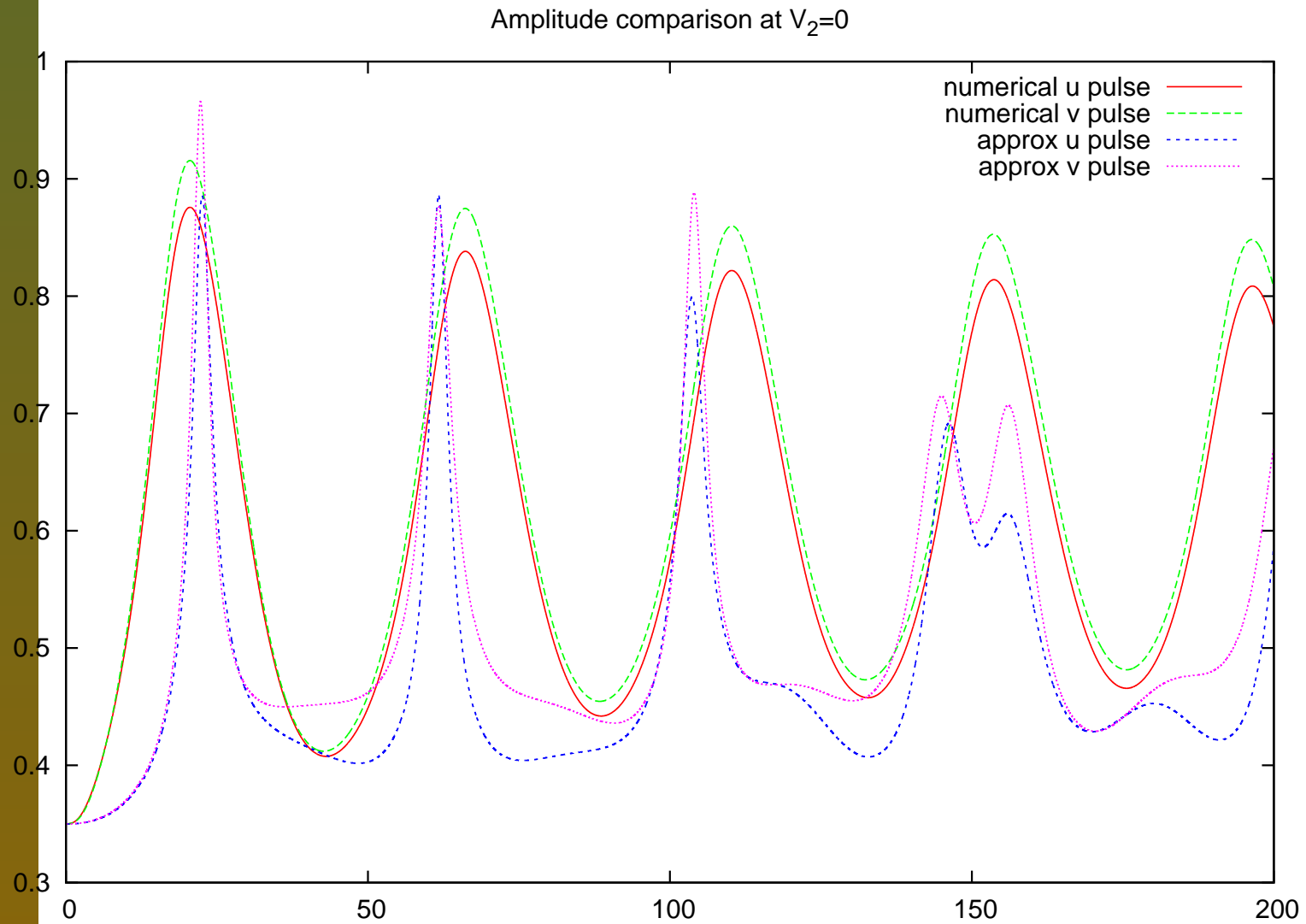
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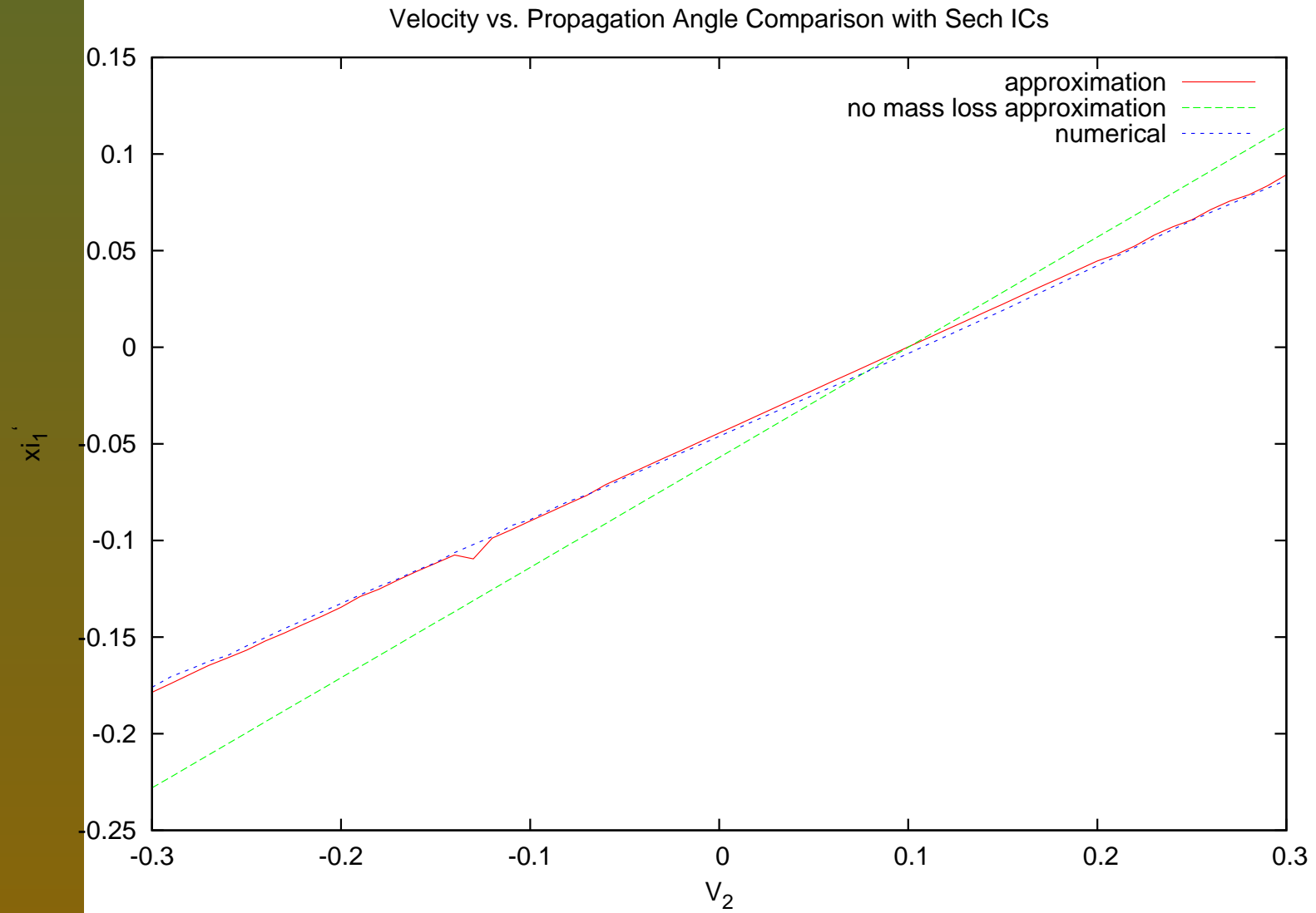
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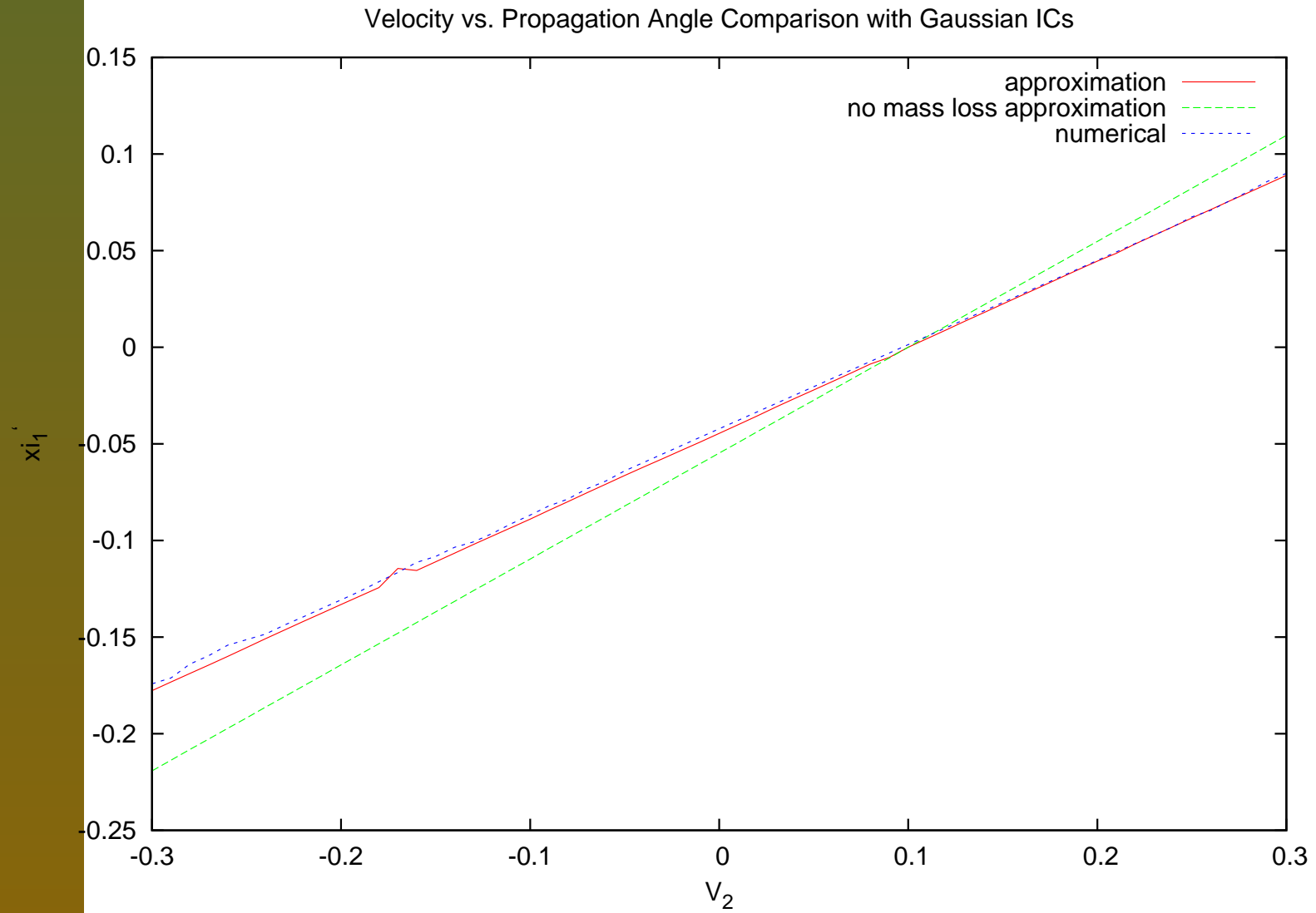
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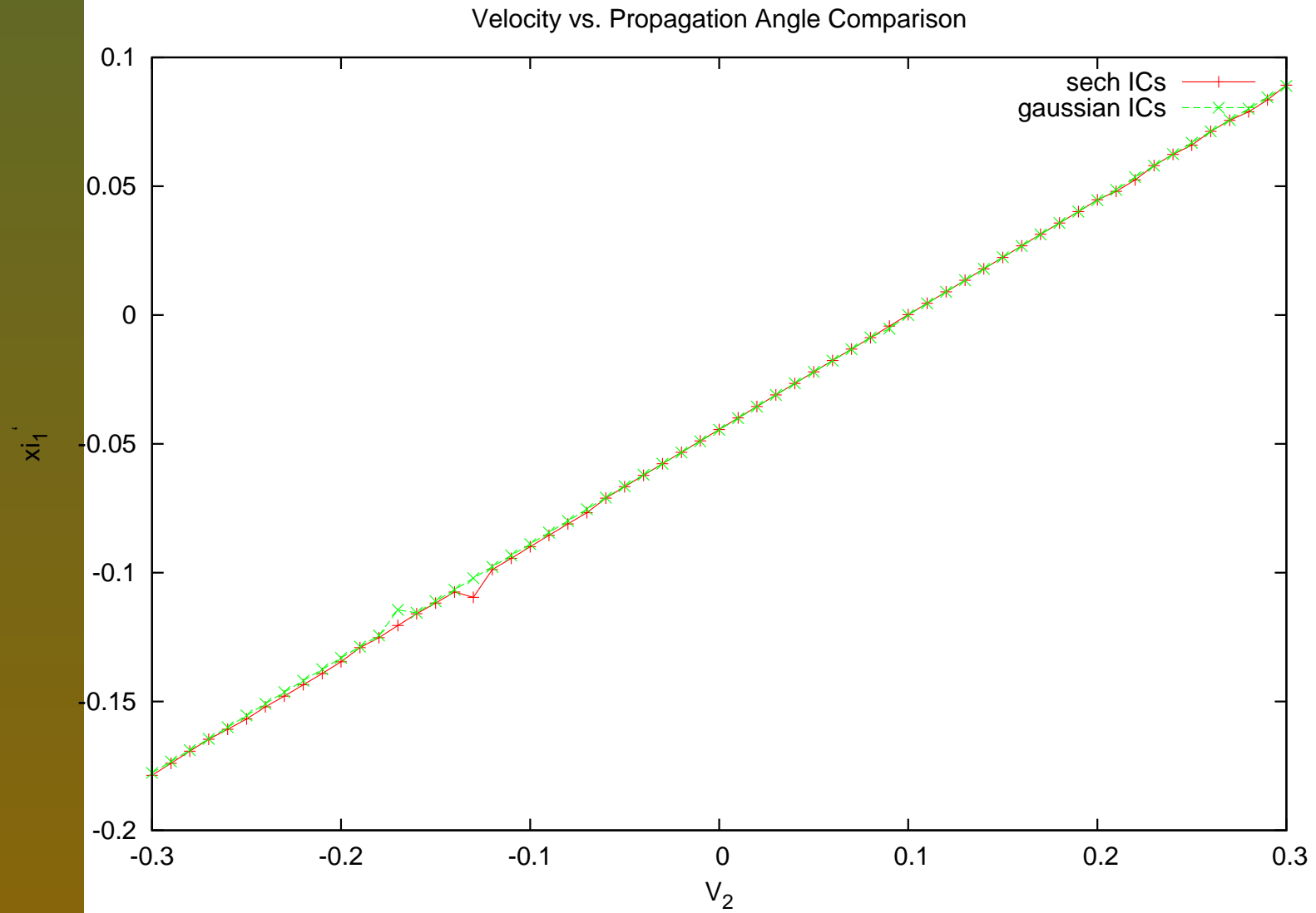
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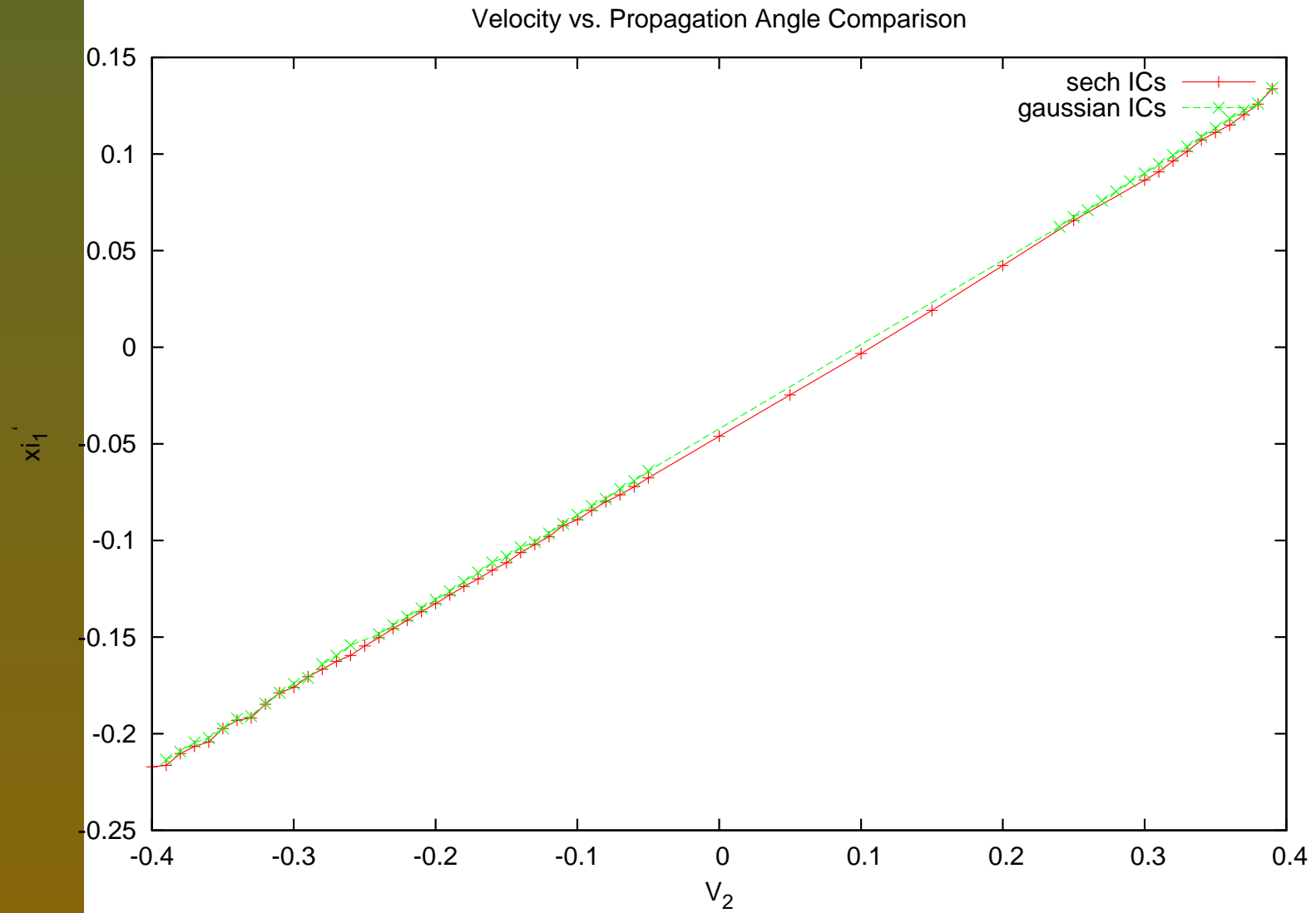
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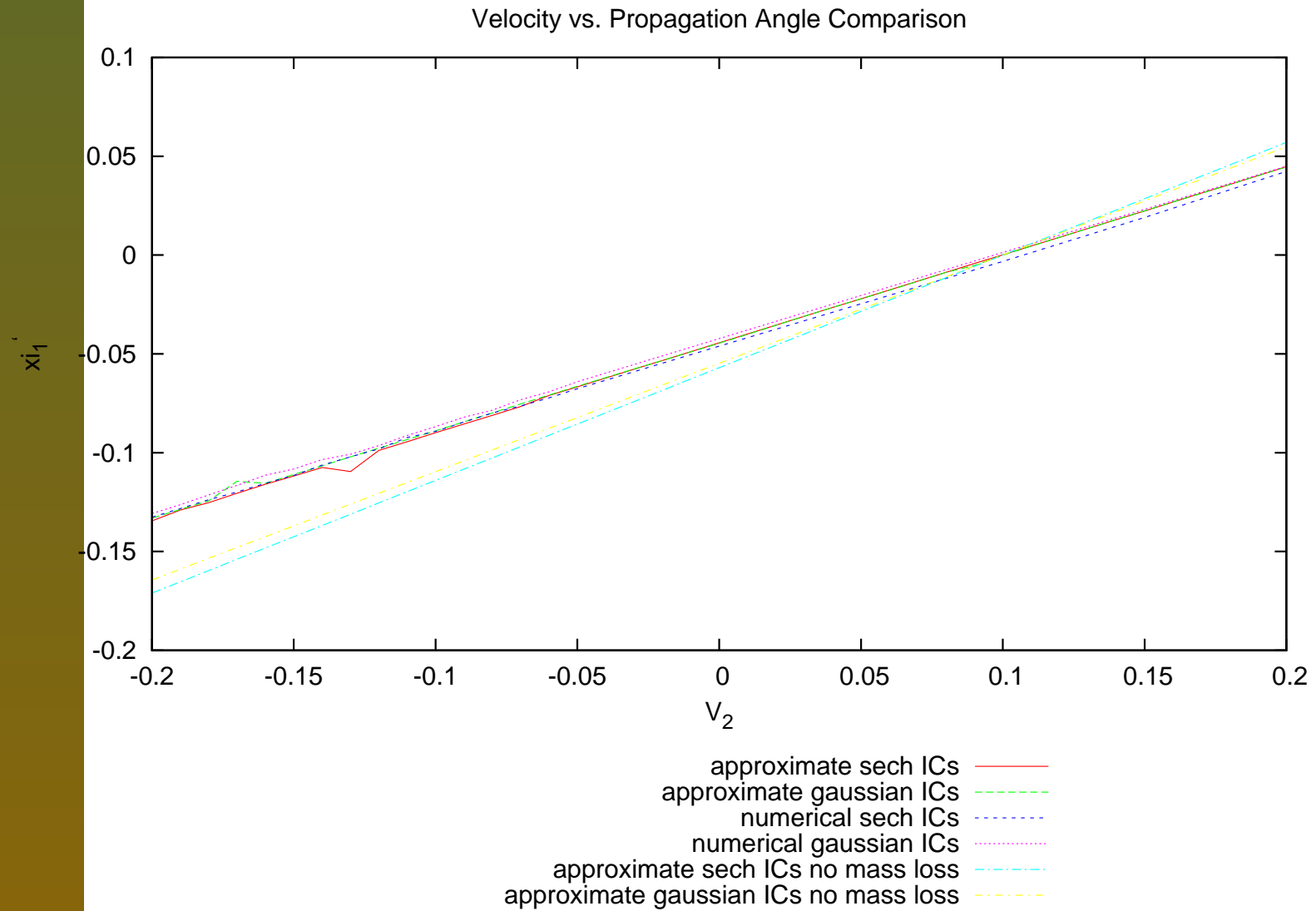
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- **Thankyou!!**